Portfolio Optimization

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The project will be carried out in English.

Introdutions

Portfolio optimization is the process of selecting the best asset distribution, optimize the weight of each asset according to some constrains and objectives. In Modern Portfolio Theory, the classical approach is called mean-variance analysis, it's a mathematical framework for assembling a portfolio of assets such that the expected return is maximized for a given level of risk.

Consider a portfolio with n asset, let $\boldsymbol{m} = \mathbb{E}[\boldsymbol{r}] = (\mathbb{E}[r_1], \mathbb{E}[r_2], \ldots \mathbb{E}[r_n])^\intercal$ be the vector of expected returns from now until one period in the future, r_i is the scalar random variable giving returns of the i^{th} asset. Let $\boldsymbol{w}$ be a n-vector weight of each asset and C be the covariance matrix of returns,

$$C = \begin{bmatrix} \sigma_1^2 & \sigma_{12} & \ldots & \sigma_{1n} \\ \sigma_{21} & \sigma_2^2 & \ldots & \sigma_{2n} \\ \ldots & & & \\ \sigma_{n1} & \sigma_{n2} & \ldots & \sigma_n^2 \end{bmatrix}$$

Therefore the portfolio risk can be express as $Var[\boldsymbol{w}^\intercal \boldsymbol{r}] = \boldsymbol{w}^\intercal C \boldsymbol{w}$, and the model can be formed as a specified mean return μ is a quadratic optimization problem:

$$\text{Minimize } \boldsymbol{w}^\intercal C \boldsymbol{w}$$

$$\text{Subject to } \boldsymbol{w}^\intercal \boldsymbol{m} = \boldsymbol{\mu} \text{ and } \boldsymbol{w}^\intercal \boldsymbol{u} = 1$$

Deliverables

Students will first derive the analytical solution of the mean-variance analysis, and then they will construct a simple portfolio with real market data. The optimization techniques will be applied to their analysis to deliver the optimal solution of their own portfolio. The code will be written in python, and a comprehensive report will be presented at the end of the project.

Work Organization

The suggested work flow is presented as follows, and will be adjusted according to students' backgrounds and interests. Additional topics may be covered if time allows.

- Introduction to Risk and Utility Theory
- Risk Metrics, Volatility, VaR (Value at Risk).
- Markowitz Efficient Frontier and mean-variance portfolio optimization.
- Inequality constrains and more approaches on optimization

References

[1] Connor, Gregory. "Active portfolio management: A quantitative approach to providing superior returns and controlling risk." (2000): 1153-1156.